MENIIT
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# JEE Advanced : Paper-2 (2014)

## IMPORTANT INSTRUCTIONS

**A. General**

1. This booklet is your Question Paper. Do not break the seal of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the left hand top corner of this sheet and on the back cover page of this booklet.
3. Blank space and blank pages are provided in the question paper for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadget of any kind are NOT allowed inside the examination hall.
5. Write your name and roll number in the space provided on the back cover of this booklet.
6. Answers to the questions and personal details are to be filled on an Optical Response Sheet, which is provided separately.
   The ORS is a doublet of two sheets – upper and lower, having identical layout. The upper sheet is a machine-gradable
   Objective Response Sheet (ORS) which will be collected by the invigilator at the end of the examination.
   The upper sheet is designed in such a way that darkening the bubble with a ball point pen will leave an identical impression at the corresponding place on the lower sheet. You will be allowed to take away the lower sheet at the end of the examination
   (**see Figure-1** on the back cover page for the correct way of darkening the bubbles for valid answers).
7. **Use a black ball point pen only to darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the lower sheet. **See Figure -1** on the back cover page for appropriate way of darkening the bubbles for valid answers.
8. DO NOT TAMPER WITH / MUTILATE THE ORS SO THIS BOOKLET.
9. On breaking the seal of the booklet check that it contains 28 pages and all the 60 questions and corresponding answer choices are legible. Read carefully the instruction printed at the beginning of each section.

**B. Filling the right part of the ORS**

10. The ORS also has a CODE printed on its left and right parts.
11. Verify that the CODE printed on the ORS (on both the left and right parts) is the same as that on the this booklet and put your signature in the Box designated as R4.
12. **IF THE CODES DO NOT MATCH, ASK FOR A CHANGE OF THE BOOKLET / ORS AS APPLICABLE.**
13. Write your Name, Roll No. and the name of centre and sign with pen in the boxes provided on the upper sheet of ORS. Do not write any of this anywhere else. Darken the appropriate bubble UNDER each digit of your Roll No. in such way that the impression is created on the bottom sheet. (**see example in Figure 2** on the back cover)

**C. Question Paper Format**

The question paper consists of **three parts** (Physics, Chemistry and Mathematics). Each part consists of two sections.

14. **Section 1** contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE is correct.**
15. **Section 2** contains **3 paragraphs** each describing theory, experiment and data etc. **Six questions** relate to three paragraphs with two questions on each paragraph. Each question pertaining to a particular passage should have **only one correct** answer among the four given choices (A), (B), (C) and (D).
16. Section 3 contains 4 multiple choice questions. **Each questions has two lists (Lits-1: P, Q, R and S; List-2, : 1, 2, 3, and 4).** The options for the correct match are provided as (A), (B), (C) and (D) out of which ONLY one is correct.

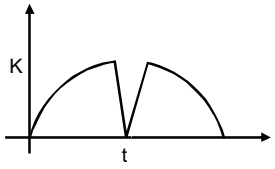
# PART A : PHYSICS

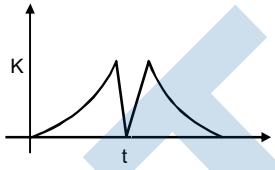
## SECTION – 1

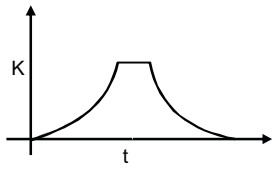
**(Only One option correct Type)**

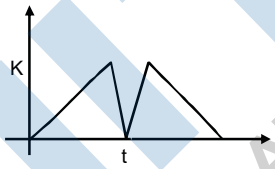
This section contains **10 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** option is correct.

**1.** A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale.

(A) 


(B*) 


(C) 

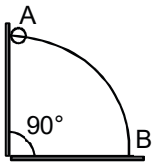

(D) 


**Ans.** B

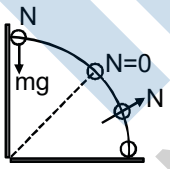
**Sol.** $K = \frac{1}{2} mv^2 = \frac{1}{2} mg^2t^2$

**2.** A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

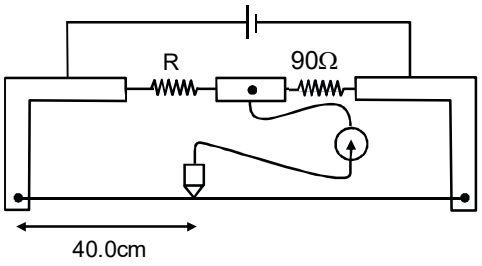
(A) always radially outwards
(B) always radially inwards
(C) radially outwards initially and radially inwards later
(D*) radially inwards initially and radially outwards later




**Ans.** D

**Sol.** 

**3.** During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is




(A) 60 ± 0.15Ω (B) 135 ± 0.56Ω (C) 60 ± 0.25Ω (D) 135 ± 0.23Ω

**Ans.** **[C]**

**Sol.** $\frac{R}{90}=\frac{40}{60}$

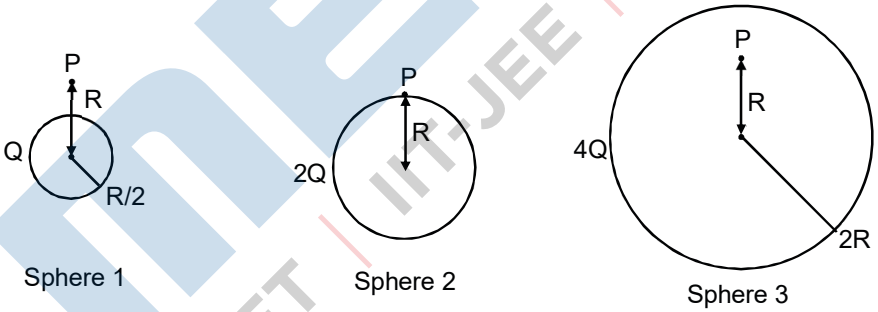
$R = 60\ \Omega$

$\frac{\Delta R}{R}=\frac{\Delta \ell}{\ell(1-\ell)}$

$\Delta R = \frac{60\times10^{-3}}{0.4\times0.6} = 0.25\ \Omega.$

**4.** Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are $E_1$, $E_2$ and $E_3$ respectively, then :

(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$ (C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$




(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$ (C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$

**Ans.** **[C]**

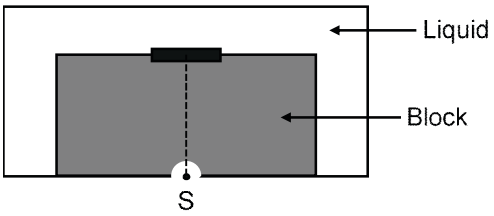
**Sol.** $\frac{kQ}{R^2}=E_1$

$E_2=\frac{k\times2Q}{R^2}=\frac{2kQ}{R^2}$

$E_3=\frac{k4Q\times R}{(2R)^3}=\frac{kQ}{2R^2}$

$E_2 > E_1 > E_3$

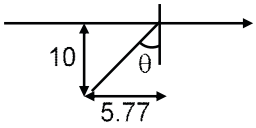
**5.** A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is




(A) 1.21 (B) 1.30 (C) 1.36 (D) 1.42

**Ans.** **[C]**

**Sol.**




$2.72 \sin\theta = n \sin 90°$

$\tan\theta = 0.577 = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

$1.36 = n$

**6.** Parallel rays of light of intensity $I = 912 Wm^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to

(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

**Ans.** **[A]**

**Sol.** $I \times \pi R^2 = 4\pi R^2 \sigma (T^4 - 300^4)$

$\frac{912}{4 \times 5.7} \times 10^9 + 300^4 = T^4$

$\Rightarrow 4 \times 10^9 + 8.1 \times 10^9 = T^4$

$121 \times 10^8 = T^4$

$\sqrt{11 \times 10^2} = T$

$T = 330\ K$

**7.** A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are $u_1$ and $u_2$, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and hc = 1240 eV nm, the work function of the metal is nearly

(A) 3.7 eV (B) 3.2 eV (C) 2.8 eV (D) 2.5 eV

**Ans.** **[A]**

**Sol.** $\frac{1}{2} m u_1^2 = \frac{1240}{248} - \phi$

$$\frac{1}{2}m\times\frac{u_1^2}{4}=\frac{1240}{310}-\phi$$

$$\frac{5}{4}-\frac{\phi}{4}=4-\phi$$

$$\frac{3\phi}{4}=\frac{4-5}{4}=\frac{11}{4}$$

$$\phi=\frac{11}{3}\text{eV}=3.66\text{eV}$$

**8.** If $\lambda_{Cu}$ is the wavelength of $K_\alpha$ X-ray line of copper (atomic number 29) and $\lambda_{Mo}$ is the wavelength of the $K_\alpha$ X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to

(A) 1.99 (B) 2.14 (C) 0.50 (D) 0.48

**Ans.** **[B]**

**Sol.** $\sqrt{f}=b(z-a)$

$$\frac{1}{\sqrt{\lambda_{Cu}}}=b(29-1)$$

$$\frac{1}{\sqrt{\lambda_{Mo}}}=b(42-1)$$

$$\sqrt{\frac{\lambda_{Mo}}{\lambda_{Cu}}}=\frac{28}{41}$$

$$\frac{\lambda_{Cu}}{\lambda_{Mo}}=\frac{41^2}{28^2}=\frac{1681}{784}\approx 2.14$$

**9.** A planet of radius $R=\frac{1}{10}\times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density $10^{-3}$ kgm$^{-1}$ into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = $6\times10^6$ m and the acceleration due to gravity on Earth is 10 ms$^{-2}$)

(A) 96 N (B) 108 N (C) 120 N (D) 150 N

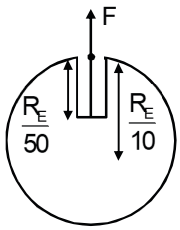
**Ans.** **[B]**

Sol. $$g=\frac{GM}{R^2}=\frac{4G}{3}\frac{\rho R^3}{R^2}=\frac{4}{3}G\rho R$$

$$g'_\rho=\frac{g}{10}=1$$

$$F=\int\lambda dyg$$

$$=10^{-3}\times\int\frac{GM}{R^2}\times\frac{y}{R}\,dy$$



$$= 10^{-3} \times 1 \left.\frac{y^2}{2R}\right|_{R/5}^{R} = \frac{10^{-3}}{2R}\left[R^2 - \frac{16R^2}{25}\right]$$

$$= \frac{9}{25} \times \frac{R}{2} \times 10^{-3}$$

$$= \frac{9}{25} \times 3 \times 10^5 \times 10^{-3} = 108 \text{ N}$$

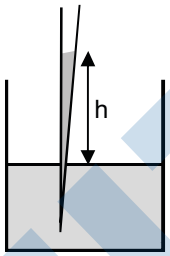
**10.** A glass capillary tube is of the shape of truncated cone with an apex angle $\alpha$ so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is $\rho$, and its contact angle with glass is $\theta$, the value of h will be (g is the acceleration due to gravity)

(A) $\frac{2s}{b\rho g}\cos(\theta - \alpha)$

(B) $\frac{2s}{b\rho g}\cos(\theta + \alpha)$

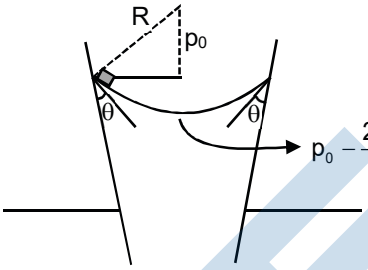
(C) $\frac{2s}{b\rho g}\cos(\theta - \alpha/2)$

(D) $\frac{2s}{b\rho g}\cos(\theta + \alpha/2)$




**Ans.** D

**Sol.**




$$p_0 - \frac{2S}{R} + h\rho g = p_0$$

$$h = \frac{2S}{R\rho g} = \frac{2S}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right)$$

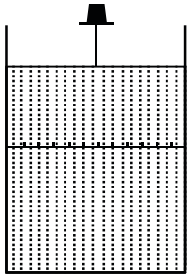
## SECTION-2

### Comprehension Type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

### Paragraph For Questions 11 & 12

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}$ R, $C_P = \frac{5}{2}$ R, and those for an ideal diatomic gas are $C_V = \frac{5}{2}$ R, $C_P = \frac{7}{2}$ R.



**11.** Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(A) 550 K (B) 525 K (C) 513 K (D) 490 K

**Ans.** **[D]**

**Sol.** $2C_p\,(T - 400) = 2\,C_v \times (700 - T)$

$$C_p = \frac{7R}{2}; C_v = \frac{3R}{2}$$

$T = 490$ K

**12.** Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

**Ans.** **[D]**

**Sol.** $n_1Cp_1\,(T - T_1) = n_2Cp_2\,(T_2 - T)$

$$2 \times \frac{7}{2} R\,(T - 400) = 2 \times \frac{5}{2} R\,(700 - T)$$

$12T = 6300$

$T = 525$ K

$W + \Delta U = 0$

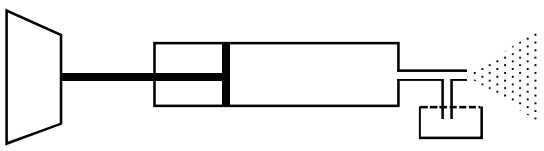
$\Delta U = n_1Cv_1\,\Delta T_1 + n_2\,Cv_2\,\Delta T_2$

$$= 2 \times \frac{3}{2} R\,(525 - 700) + 2 \times \frac{5}{2} R \times (525 - 400) = -525R + 625\,R = 100\,R$$

$W = -100R$

**Paragraph For Questions 13 & 14**

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



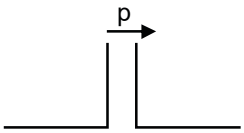
**13.** If the piston is pushed at a speed of 5 $mms^{-1}$, the air comes out of the nozzle with a speed of

(A) 0.1 $ms^{-1}$ (B) 1 $ms^{-1}$ (C) 2 $ms^{-1}$ (D) 8 $ms^{-1}$

**Sol.** $A_1v_1 = A_2v_2$

$\pi \times 20^2 \times 5 = \pi \times 1^2 \times v_2$

$v_2 = 2000$ mm/s = 2 m/s




**14.** If the density of air is $\rho_a$ and that of the liquid $\rho_\ell$, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a\rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) $\rho_\ell$

**Ans.** A

**Sol.** $\frac{p_0}{\rho_a g} + 0 = \frac{p}{\rho_a g} + \frac{v^2}{2g}$

$p = p_0 - \frac{1}{2}\rho_a v^2$

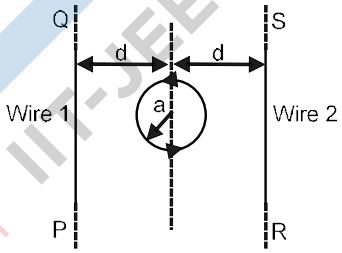
$\Delta p = \frac{1}{2}\rho_a v^2$

$\frac{p_0 - \Delta p}{\rho_\ell g} + \frac{v_\ell^2}{2g} = \frac{p_0 - \rho_\ell gh}{\rho_\ell g}$

$v_\ell = \sqrt{2gh + \frac{\rho_a}{\rho_\ell}v^2}$

**Paragraph For Questions 15 & 16**

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.




**15.** When d ≈ a but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ a

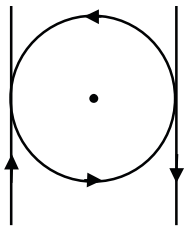
(B) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ a

(C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ 1.2a

(D) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ 1.2a

**Ans.** **[C]**

**Sol.** $\frac{\mu_0 i \times a^2}{2(a^2+h^2)^{3/2}} = \frac{\mu_0 i \times a \times 2}{2\pi\left(\sqrt{h^2+a^2}\right)^2}$



$\Rightarrow \frac{\pi a}{2} = \sqrt{a^2+h^2}$

$\frac{\pi^2}{4}a^2 - a^2 = h^2$

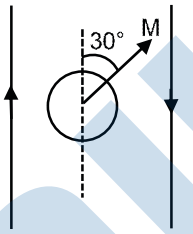
$h = \sqrt{1.5}\ a \approx 1.2\ a$

**16.** Consider d >> a, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$ (C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

**Ans.** **[B]**

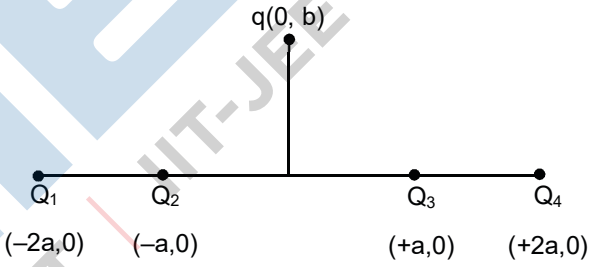
**Sol.** $\tau = \vec{M} \times \vec{B}$




$B = \frac{\mu_0 i}{2\pi d} \times 2$

$\tau = i\pi a^2 \times \frac{\mu_0 i}{\pi d} \sin 150° = \frac{\mu_0 i^2 a^2}{2d}$

**17.** Four charges $Q_1$, $Q_2$, $Q_3$ and $Q_4$ of same magnitude are fixed along the x axis at x = –2a, –a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List-I. The direction of the forces on the charge q is given in List-II. Match List-I with List-II and select the correct answer using the code given below the lists.




| | List-I | List-II | |
|---|---|---|---|
| **P.** | $Q_1$, $Q_2$, $Q_3$, $Q_4$ all positive | **1.** | +x |
| **Q.** | $Q_1$, $Q_2$ positive; $Q_3$, $Q_4$ negative | **2.** | –x |
| **R.** | $Q_1$, $Q_4$ positive; $Q_2$, $Q_3$ negative | **3.** | +y |
| **S.** | $Q_1$, $Q_3$ positive; $Q_2$, $Q_4$ negative | **4.** | –y |

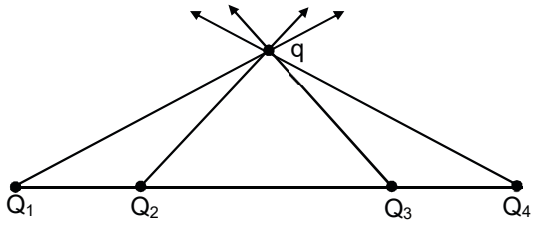
**Code :**

(A) P-3, Q-1, R-4, S-2 (B) P-4, Q-2, R-3, S-1

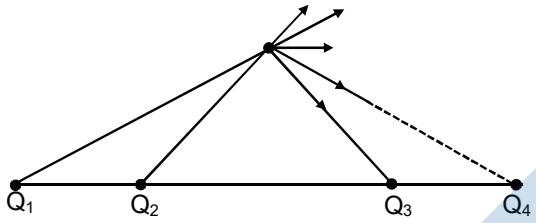
(C) P-3, Q-1, R-2, S-4 (D) P-4, Q-2, R-1, S-3

**Ans.** **[A]**

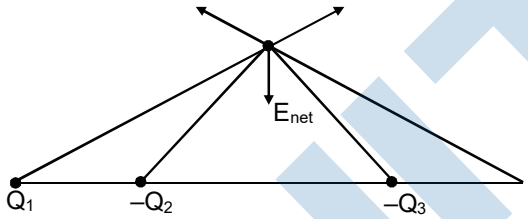
**Sol.** P




Q → 1




R → 4




**18.** Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists.

| **List-I** | | **List-II** | |
|---|---|---|---|
| **P.** |  | **1.** | 2r |
| **Q.** |  | **2.** | r/2 |
| **R.** |  | **3.** | –r |
| **S.** |  | **4.** | r |

Code :

(A) P-1, Q-2, R-3, S-4 (B) P-2, Q-4, R-3, S-1

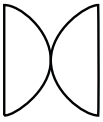
(C) P-4, Q-1, R-2, S-3 (D) P-2, Q-1, R-3, S-4

**Ans.** **[B]**

**Sol.** $\frac{1}{8} = (1.5-1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{1}{R}$

P $\frac{1}{f_{eq}} = \frac{2}{R} \Rightarrow f_{eq} = R/2$

$P \rightarrow 2$



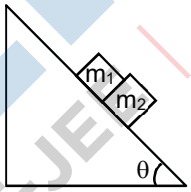
$\frac{1}{f} = (1.5-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{1}{2R}$

$\frac{1}{f_{eq}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R}$

$Q \rightarrow 4$

**19.** A block of mass $m_1$ = 1 kg another mass $m_2$ = 2kg, are placed together (see figure) on an inclined plane with angle of inclination $\theta$. Various values of $\theta$ are given in List-I. The coefficient of friction between the block $m_1$ and the plane is always zero. The coefficient of static and dynamic friction between the block $m_2$ and the plane are equal to $\mu = 0.3$. In List-II expressions for the friction on block $m_2$ are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by g.

[Useful information : tan (5.5°) 0.1; tan (11.5°) 0.2 ; tan (16.5°) 0.3]




| | List-I | | List-II |
|---|---|---|---|
| **P.** | $\theta = 5°$ | **1.** | $m_2 g \sin\theta$ |
| **Q.** | $\theta = 10°$ | **2.** | $(m_1 + m_2) g \sin\theta$ |
| **R.** | $\theta = 15°$ | **3.** | $\mu m_2 g \cos\theta$ |
| **S.** | $\theta = 20°$ | **4.** | $\mu(m_1 + m_2) g \cos\theta$ |

**Code :**

(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3

(C) P-2, Q-2, R-2, S-4 (D) P-2, Q-2, R-3, S-3

**Ans.** **[D]**

**Sol.** $(m_1 + m_2) g \sin\theta = 3 \times g \times \sin 5° \approx 3g \times \frac{5\pi}{180}$

$f_{max} = 0.3 \times m_2 g \cos 5°$

$0.6\ g \cos 5° \approx 0.6\ g$

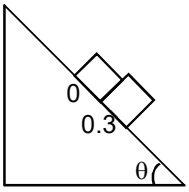
$\Rightarrow f = (m_1 + m_2)\ g \sin\theta$

sliding at

$(m_1 + m_2)\ g \sin\theta = \mu\ m_2 g \cos\theta$

$\tan\theta = \frac{0.3\times 2}{1+2} = 0.2$

$\theta = 11.5°$




$P \to 2 \quad Q \to 2 \quad R \to 3 \quad S \to 3$

**20.** A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List-1 and the distance where the water jet hits the floor of the lift is given in List-II. Match the statements from List-I with those in List-II and select the correct answer using the code given below the lists.

| | **List-I** | | **List-II** |
|---|---|---|---|
| P. | Lift is acceleration vertically up. | 1. | d = 1.2 m |
| Q. | Lift is accelerating vertically down with an acceleration less than the gravitational acceleration | 2. | d > 1.2 m |
| R. | Lift is moving vertically up with constant speed. | 3. | d < 1.2 m |
| S. | Lift is falling freely. | 4. | No water leaks out of the jar |

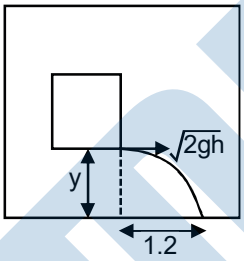
**Code :**

(A) P-2, Q-3, R-2, S-4 (B) P-2, Q-3, R-1, S-4

(C) P-1, Q-1, R-1, S-4 (D) P-2, Q-3, R-1, S-1

**Ans.** C

**Sol.**




$y = \frac{1}{2} g t^2$

$t = \sqrt{\frac{2y}{g}}$

$d = \sqrt{2gh} \times \sqrt{\frac{2y}{g}} = 2\sqrt{yh}$

$P \to 1 \quad Q \to 1 \quad R \to 1\ S \to 4$

# PART B : CHEMISTRY

## SECTION - 1

**(Only One option correct Type)**

This section contains **10 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** option is correct.

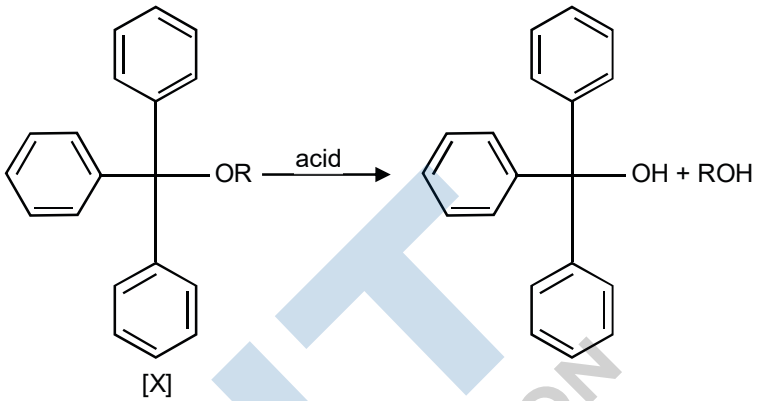
**21.** The acidic hydrolysis of ether (X) shown below is fastest when

(A) one phenyl group is replaced by a methyl group.

(B) one phenyl group is replaced by a para-methoxyphenyl group.

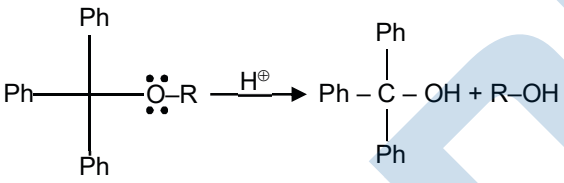
(C) two phenyl groups are replaced by two para-methoxyphenyl groups.

(D) no structural change is made to X.

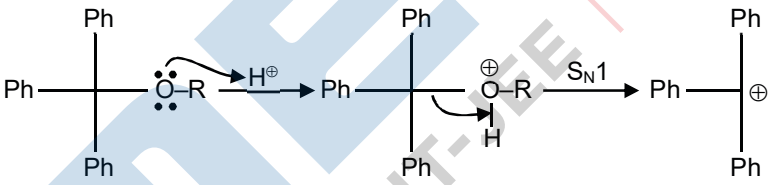


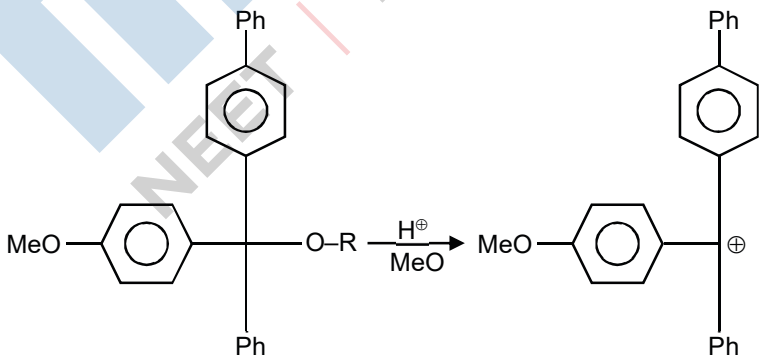

**Ans.** **[C]**

**Sol.**



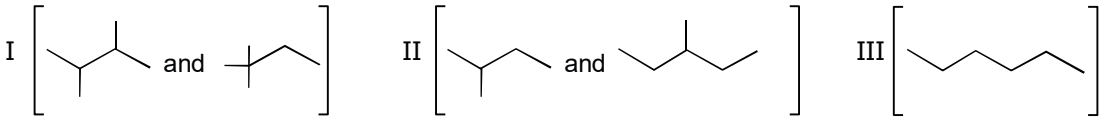

Two parametoxy phenyl replace. Two phenyl group on basis of $S_N1$ mechanism and so stability of carbocation favour.




more stable due to +M effect of MeO–

**22.** Isomers of hexane, based on their brancing, can be divided into three distinct classes as shown in the figure.



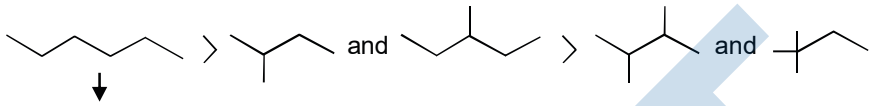

The correct order of their boiling point is

(A) I > II > III (B) III > II > I (C) II > III > I (D) III > I > II

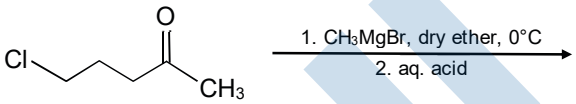
**Ans.** **[B]**

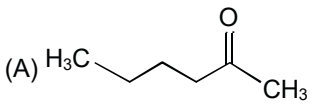
**Sol.** B.P. $\propto$ Surface area

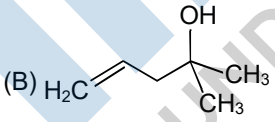
III > II > I

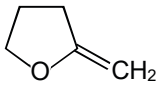


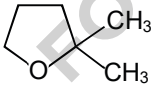

**23.** The major product in the following reaction is



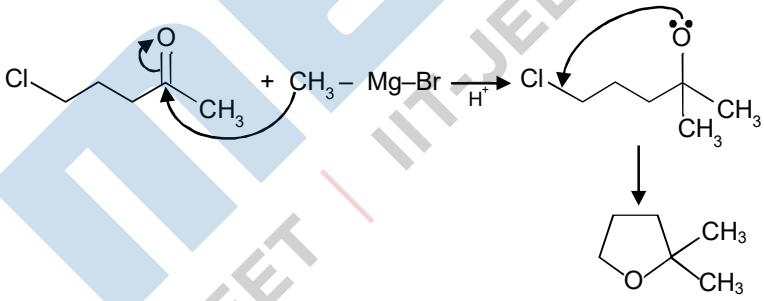

(A) 


(B) 


(C) 


(D) 


**Ans.** **[D]**

**Sol.** 


**24.** Hydrogen peroxide in its reaction with $KIO_4$ and $NH_2OH$ respectively, is acting as a

(A) reducing agent, oxidising agent (B) reducing agent, reducing agent

(C) oxidising agent, oxidising agent (D) oxidising agent, reducing agent

**Ans.** **[A]**

**Sol.** $KIO_4 + H_2O_2 \longrightarrow KIO_3 + H_2O + O_2$

$NH_2–OH + H_2O_2 \longrightarrow N_2 + 4H_2O$

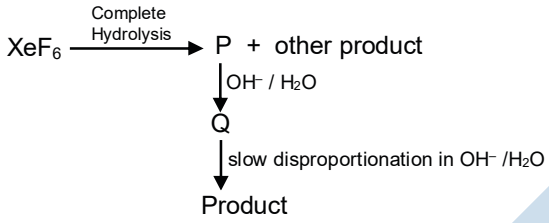
**25.** The product formed in the reaction of $SOCl_2$ with white phosphorous is

(A) $PCl_3$ (B) $SO_2Cl_2$ (C) $SCl_2$ (D) $POCl_3$

**Ans.** **[A]**

**Sol.** $P_4 + SOCl_2 \longrightarrow PCl_3 + SO_2 + S_2Cl_2$

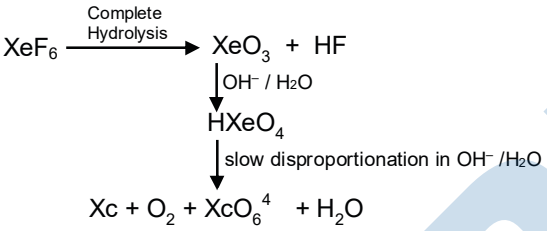
**26.** Under ambient conditions, the total number of gases released as products in the final step of the reaction scheme shown below is




(A) 0 (B) 1 (C) 2 (D) 3

**Ans.** **[C]**

**Sol.**




**27.** For the identification of β-naphthol using dye test, it is necessary to use

(A) dichloromethane solution of β-naphthol (B) acidic solution of β-naphthol

(C) neutral solution of β-naphthol (D) alkaline solution of β-naphthol

**Ans.** **[D]**

**Sol.** In alkaline solution it exists as anion which is very strongly activated towards AES reaction so dye formation takes place.

**28.** For the elementary reaction M → N, the rate of disappearance of M increases by a factor of 8 upon doubling the concentration of M. The order of the reaction with respect to M is

(A) 4 (B) 3 (C) 2 (D) 1

**Ans.** **[B]**

**Sol.** rate of Reaction = $K[M]^x$

$$\frac{r_1}{r_2} = \frac{K[M]^x}{K[2M]^x}$$

$$\frac{1}{8} = \frac{1}{2^x}$$

$x = 3$

**Note :** *Here either this reaction is not elementary or if it is elementary then not balanced.*

**29.** For the process

$$H_2O(\ell) \longrightarrow H_2O(g)$$

at T = 100°C and 1 atmosphere pressure, the correct choice is

(A) $\Delta S_{system} > 0$ and $\Delta S_{surroundings} > 0$ (B) $\Delta S_{system} > 0$ and $\Delta S_{surroundings} < 0$

(C) $\Delta S_{system} < 0$ and $\Delta S_{surroundings} > 0$ (D) $\Delta S_{system} < 0$ and $\Delta S_{surroundings} < 0$

**Ans.** **[B]**

**Sol.** $H_2O(\ell) \longrightarrow H_2O(g)$

For system

Liquid is getting converted into gas $\boxed{\Delta S_{system} > 0}$

Process is endothermic $\Rightarrow$ Heat is given by system

$$\Delta S_{surrounding} = \frac{q_{surr}}{T}$$

where $q_{surrounding} = -\Delta H$

$\because$ $q_{surr} < 0$

$\Delta S_{surr} < 0$

**30.** Assuming 2s-2p mixing in NOT operative, the paramagnetic species among the following is

(A) $Be_2$ (B) $B_2$ (C) $C_2$ (D) $N_2$

**Ans.** **[C]**

**Sol.** $C_2 \rightarrow (Be)\ (\sigma\ 2p_z^2)\ (\pi 2p_x^1\ \pi 2p_y^1)$

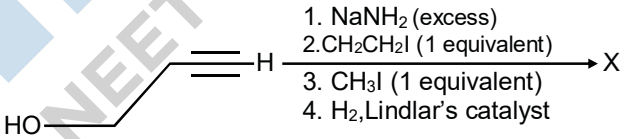
## SECTION – 2

### Comprehension Type (Only One Option Correct)

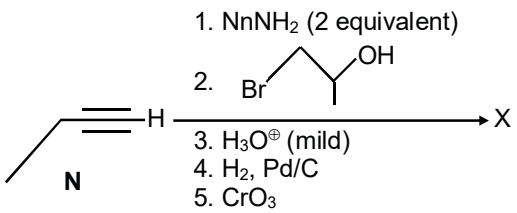
This section contains **3 paragraphs** each describing theory, experiment, data etc. **Six questions** relate to three paragraphs with two questions on each paragraph. Each question has **only one correct answer** among the four given options (A), (B), (C) and (D).

### Paragraph For Questions 31 to 32

Schemes 1 and 2 describe sequential transformation of alkynes M and N. Consider only the major products formed in each step for both the schemes.

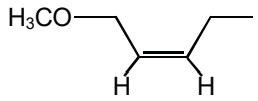


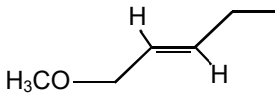

**Scheme–1**

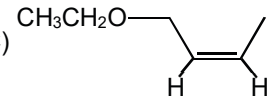


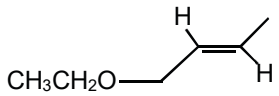

**Scheme–2**

**31.** The product X is

(A) 

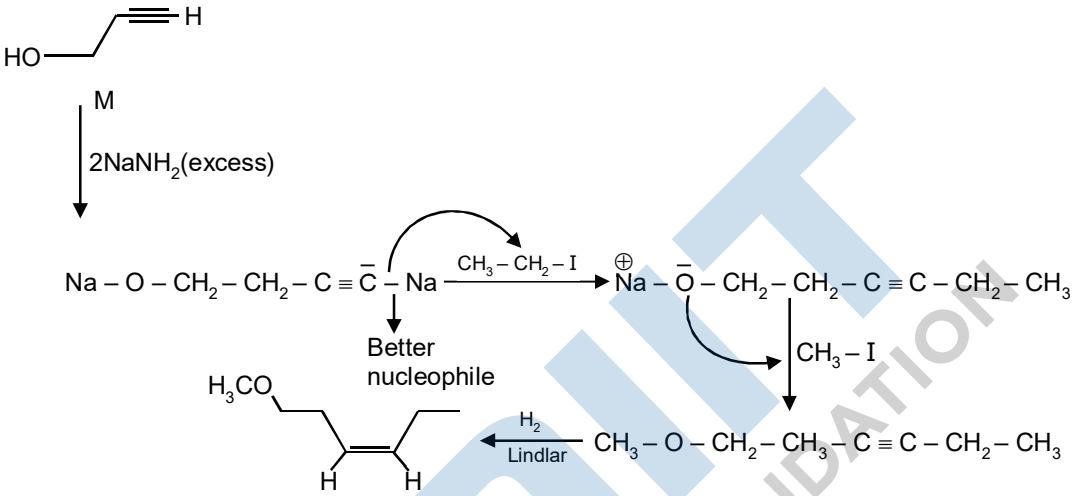
(B) 

(C) 

(D) 

**Ans.** **[A]**

**Sol.**




**32.** The correct statement with respect to product Y is

(A) It gives a positive Tollens test and is functional isomer of X

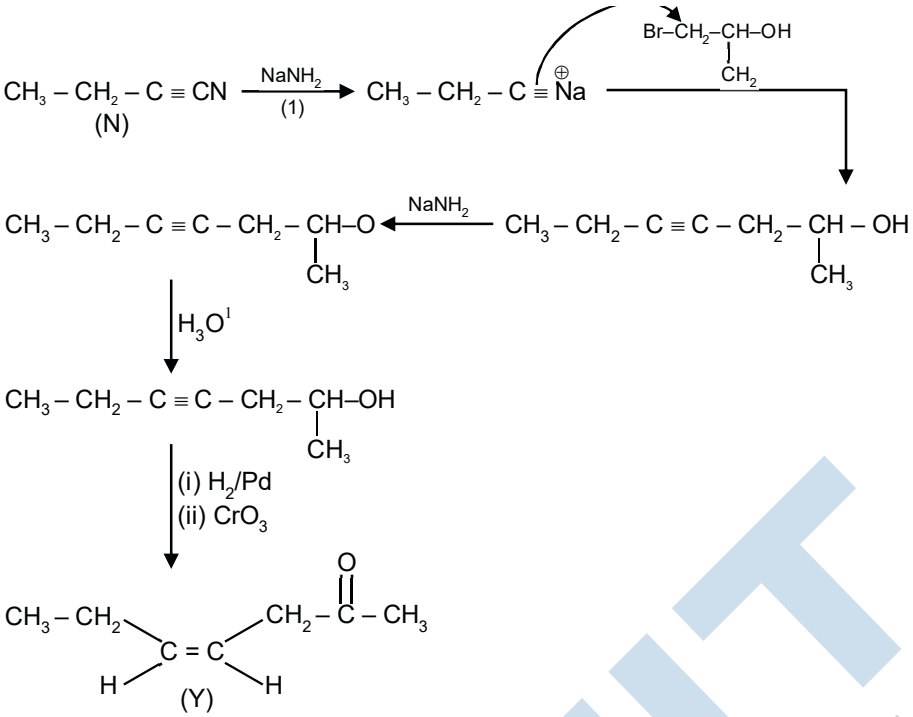
(B) It gives a positive Tollens test and is a geometrical isomer of X

(C) It gives a positive iodoform test and is a functional isomer of X.

(D) it gives a positive iodoform test and is a geometrical isomer of X.

**Ans.** **[C]**

**Sol.**




Gives +ve Idoform test

**Paragraph for Questions 33 and 34**

An aqueous solution of metal ion **M1** reacts separately with reagents **Q** and **R** in excess to give tetrahedral and square planar complexes, respectively. An aqueous solution of another metal ion **M2** always forms tetrahedral complexes with these reagents. Aqueous solution of **M2** on reaction with reagent **S** gives white precipitate which dissolves in excess of **S**. The reactions are summarized in the scheme given below :

**SCHEME :**

Tetrahedral $\xleftarrow[\text{excess}]{Q}$ M1 $\xrightarrow[\text{excess}]{R}$ Square planar

Tetrahedral $\xleftarrow[\text{excess}]{Q}$ M2 $\xrightarrow[\text{excess}]{R}$ Tetrahedral

M2 $\xrightarrow{S,\ \text{stoichiometric amount}}$ White precipitate $\xrightarrow[\text{excess}]{S}$ precipitate dissolves

**33.** M1, Q and R respectively are

(A) $Zn^{2+}$, KCN and HCl (B) $Ni^{2+}$, HCl and KCN

(C) $Cd^{2+}$, KCN and HCl (D) $Co^{2+}$, HCl and KCN

**Ans.** **[B]**

**Sol.** $[NiCl_4]^{2-}$ Tetrahedral $\xleftarrow{HCl}$ $Ni^{2+}$ $\xrightarrow{KCN(excess)}$ Square planar $[Ni(CN)_4]^{2-}$

$[ZnCl_4]^{2-}$ Tetrahedral $\xleftarrow[\text{excess}]{HCl}$ $Zn^{2+}$ $\xrightarrow[\text{excess}]{KCN}$ Tetrahedral $[Zn(CN)_4]^{2-}$

$Zn^{2+} \xrightarrow{KOH,\ \text{stoichiometric amount}} Zn(OH)_2$ White precipitate $\xrightarrow[\text{excess}]{S}$ $K_2ZnO_2$ precipitate dissolves

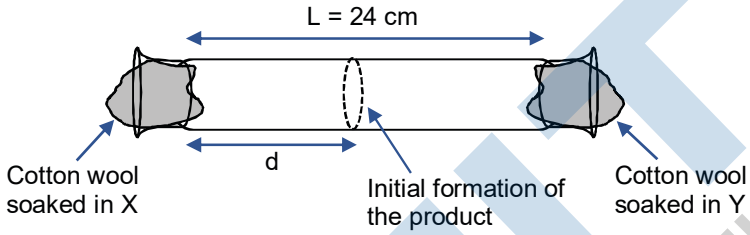
**34.** Reagent S is

(A) $K_4[Fe(CN)_6]$ (B) $Na_2HPO_4$ (C) $K_2CrO_4$ (D)KOH

**Ans.** **[D]**

**Paragraph for Questions 35 and 36**

**X** and **Y** are two volatile liquids with molar weights of 10 g $mol^{-1}$ and 40 g $mol^{-1}$ respectively. Two cotton plugs, one soaked in **X** and the other soaked in **Y,** are simultaneously placed at the ends of a tube of length **L** = 24 cm, as shown in the figure. The tube is filled with an inert gas at 1 atmosphere pressure and a temperature of 300 K. Vapours of **X** and **Y** react to form a product which is first observed at a distance **d** cm from the plug soaked in **X**. Take **X** and **Y** to have equal molecular diameters and assume ideal behaviour for the inert gas and the two vapours.




**35.** The value of d in cm (shown in the figure), as estimated from Graham's law, is

(A) 8 (B) 12 (C) 16 (D) 20

**Ans.** **[C]**

**Sol.** $\frac{r_x}{r_y} = \sqrt{\frac{40}{10}} = \sqrt{\frac{M_y}{M_x}}$

$\frac{x}{24-x} = 2$

$x = 48 - 2x$

$x = 16$

**36.** The experimental value of d is found to be smaller than the estimate obtained using Graham's law. This is due to

(A) larger mean free path for X as compared to that of Y

(B) larger mean free path for Y as compared to that of X.

(C) increased collision frequency of Y with the inert gas as compared to that of X with the inert gas.

(D) increases collision frequency of X with the inert gas as compared to that of Y with the inert gas.

**Ans.** **[D]**

**Sol.** Mean free math $(\lambda) = \frac{1}{\sqrt{2}\pi\sigma^2 N^*}$

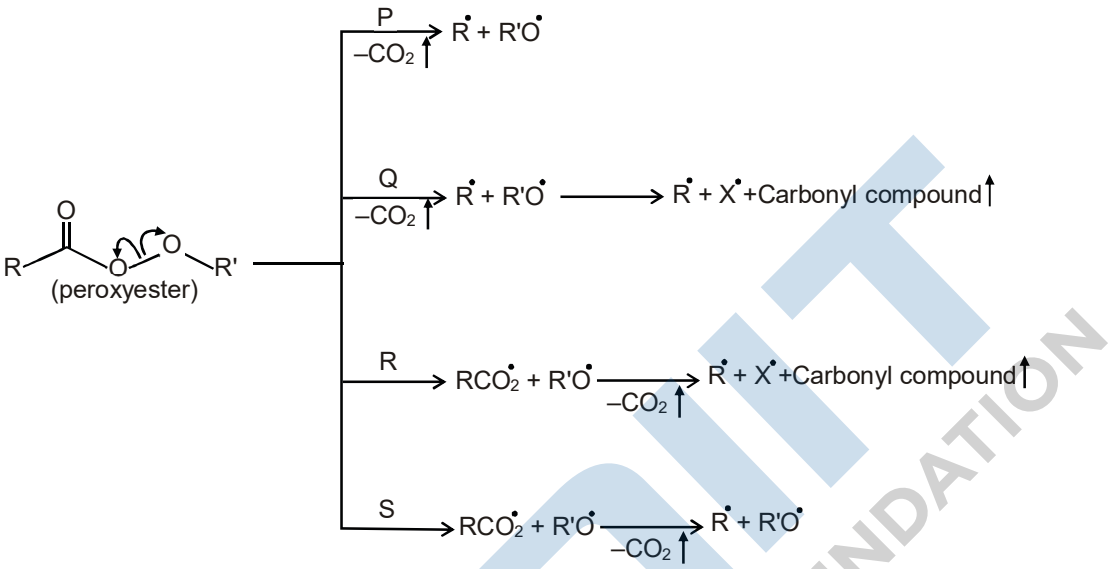
Here $\sigma$ is same but no information is given about N*, so we are considering it same for both, therefore mean free path is same for both gases and collision frequency of gas X will be higher than gas Y & it will travel less distance than expected. Out of all given option D is best answer.

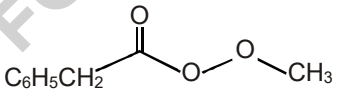
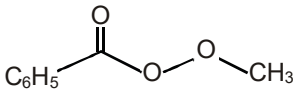
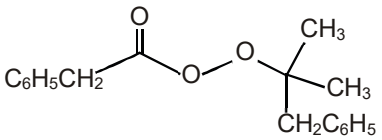
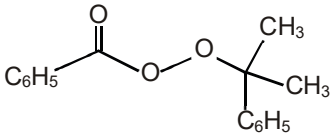
## SECTION – 3

**Matching List Type (Only One Option Correct)**

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D), out of which one is correct.

**37.** Different possible thermal decomposition pathways for peroxyesters are shown below. Match each pathway from List I with an appropriate structure from List II and select the correct answer using the code given below the lists.




| List-I | | List-II | |
|---|---|---|---|
| **p.** | Pathway P | **1.** |  |
| **Q.** | Pathway Q | **2.** |  |
| **R.** | Pathway R | **3.** |  |
| **S.** | Pathway S | **4.** |  |

**Codes:**

| | P | Q | R | S | | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|---|
| (A) | 1 | 3 | 4 | 2 | (B) | 2 | 4 | 3 | 1 |
| (C) | 4 | 1 | 2 | 3 | (D) | 3 | 2 | 1 | 4 |

**Ans.** **[A]**

**Sol.** (1) $C_6H_5CH_2-\underset{(P)}{\overset{O}{\overset{\|}{C}}}-O-O-Me \xrightarrow[CO_2(\uparrow)]{\Delta} C_6H_5-\dot{C}H_2 + MeO^{\circ}$ (R), $R'-O^{\circ}$

(2) $C_6H_5-\underset{(S)}{\overset{O}{\overset{\|}{C}}}-O-O-CH_3 \xrightarrow{\Delta} C_6H_5-\overset{O}{\overset{\|}{C}}-O^{\circ} + CH_3-O^{\circ}$

$(RCO_2^{\circ})$ $(R'-O^{\circ})$

$C_6H_5 + CO_2(\ell)$

$(R^{\circ})$

(3) $C_6H_5CH_2-\underset{(Q)}{\overset{O}{\overset{\|}{C}}}-O-\underset{CH_2C_6H_5}{C}-CH_3 \xrightarrow[-CO_2]{\Delta} C_6H_5\dot{C}H_2 + CH_3-\underset{CH_2C_6H_5}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}}-O^{\circ}$

$(R^{\circ})$ $(R'O^{\circ})$

$\xrightarrow{\Delta}$ $Me_2C=O + C_6H_5CH_2$ $(R^{\circ})$

(4) $C_6H_5-C(CH_3)(C_6H_5^{\circ})-O-O-C(=O)-CH_3 \xrightarrow{\Delta} C_6H_5-C(=O)-O^{\circ} + CH_3-C(C_6H_5)-O$

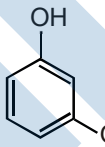
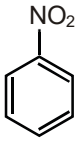
$(RCO_2^{\circ})$ $(R'-O^{\circ})$

$C_6H_5^{\circ}$ $(R^{\circ})$

$(CH_3)_2C=O + C_6H_5^{\circ}$ $(X^{\circ})$

(carbonyl compound)

**38.** Match the four starting materials **(P, Q, R, S)** given in List I with the corresponding reaction schemes **(I, II, III, IV)** provided in List II and select the correct answer using the code given below the lists.

| **List–I** | | **List–II** |
|---|---|---|
| **P.** | H—≡—H | **1. Scheme I** |
| **Q.** |  | **2. Scheme II** |
| **R.** |  | **3. Scheme III** |

**Scheme I**

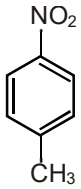
$? \xrightarrow{(i)\ KMnO_4, HO^{\ominus}, \text{heat}\ (ii)\ H^{\oplus}, H_2O \ (iii)\ SOCl_2\ (iv)\ NH_3} C_7H_6N_2O_3$

**Scheme II**

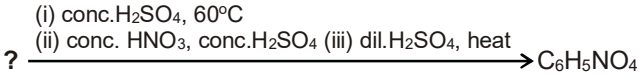
$? \xrightarrow{(i)\ Sn/HCl\ (ii)\ CH_3COCl\ (iii)\ \text{conc.}\ H_2SO_4 \ (iv)\ HNO_3\ (v)\ \text{dil.}\ H_2SO_4\ (vi)\ HO^{\ominus}} C_6H_6N_2O_2$

**Scheme III**

$? \xrightarrow{(i)\ \text{red hot iron, 873 K}\ (ii)\ \text{fu min g}\ HNO_3, H_2SO_4, \text{heat} \ (iii)\ H_2S.NH_3\ (iv)\ NaNO_2, H_2SO_4\ (v)\ \text{hydrolysis}} C_6H_5NO_3$

**S.** 

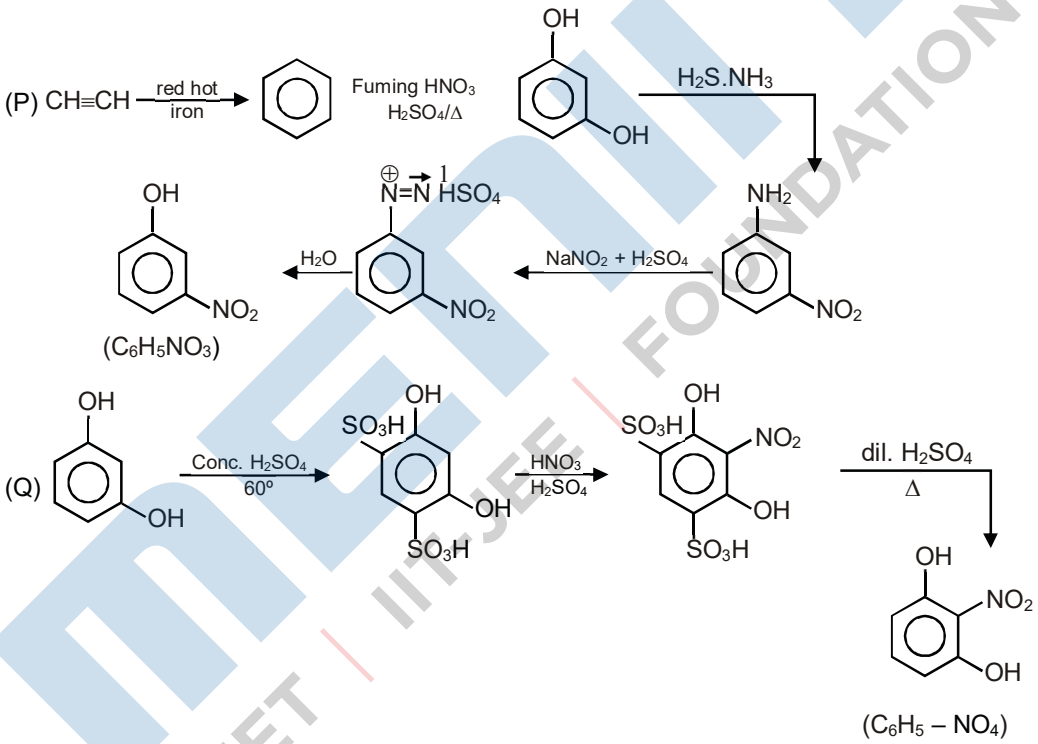
**4. Scheme IV**

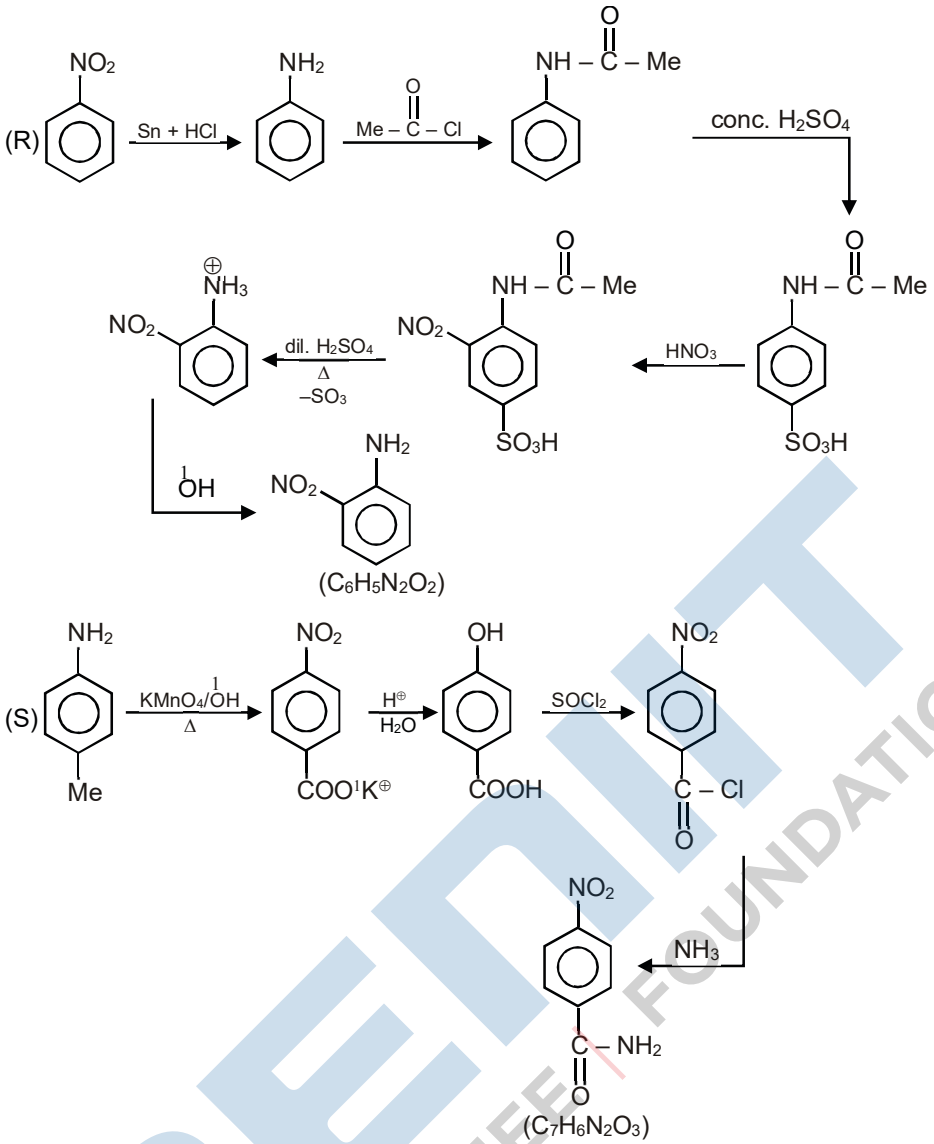



**Code:**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 1 | 4 | 2 | 3 |
| (B) | 3 | 1 | 4 | 2 |
| (C) | 3 | 4 | 2 | 1 |
| (D) | 4 | 1 | 3 | 2 |

**Ans.** **[C]**

**Sol.** 




**39.** Match each coordination compound in List-I with an appropriate pair of characteristics from List-II and select the correct answer using the code given below the lists.

{en = $H_2NCH_2CH_2NH_2$; atomic numbers : Ti = 22; Cr = 24; Co = 27; Pt = 78}

| | List-I | List-II |
|---|---|---|
| **P.** | $[Cr(NH_3)_4Cl_2]Cl$ | **1.** Paramagnetic and exhibits ionisation isomerism. |
| **Q.** | $[Ti(H_2O)_5Cl](NO_3)_2$ | **2.** Diamagnetic and exhibits cis-trans isomerism. |
| **R.** | $[Pt(en)(NH_3)Cl]NO_3$ | **3.** Paramagnetic and exhibits cis-trans isomerism. |
| **S.** | $[Co(NH_3)_4(NO_3)_2]NO_3$ | **4.** Diamagnetic and exhibits ionisation isomerism. |

**Code:**

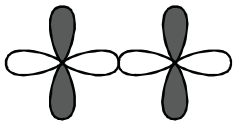
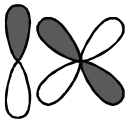
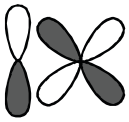
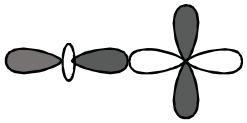
| | P | Q | R | S | | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|---|
| (A) | 4 | 2 | 3 | 1 | (B) | 3 | 1 | 4 | 2 |
| (C) | 2 | 1 | 3 | 4 | (D) | 1 | 3 | 4 | 2 |

**Ans.** **[B]**

**Sol.**

| | |
|---|---|
| $[Cr(NH_3)_4Cl_2]Cl$ | Paramagnetic and exhibits cis-trans isomerism. |
| $[Ti(H_2O)_5Cl](NO_3)_2$ | Paramagnetic and exhibits ionisation isomerism. |
| $[Pt(en)(NH_3)Cl]NO_3$ | Diamagnetic and exhibits ionisation isomerism. |
| $[Co(NH_3)_4(NO_3)_2]NO_3$ | Diamagnetic and exhibits cis-trans isomerism. |

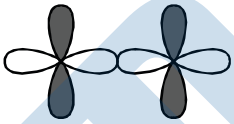
**40.** Match the orbital overlap figures shown in List-I with the description given in List-II and select the correct answer using the code given below the lists.

| **List-I** | | **List-II** |
|---|---|---|
| (P) |  | (1) p – d p antibonding |
| (Q) |  | (2) d – d s bonding |
| (R) |  | (3) p – d p bonding |
| (S) |  | (4) d – d s antibonding |

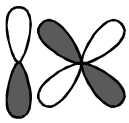
**Code :**

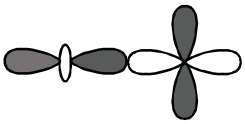
| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 2 | 1 | 3 | 4 |
| (B) | 4 | 3 | 1 | 2 |
| (C*) | 2 | 3 | 1 | 4 |
| (D) | 4 | 1 | 3 | 2 |

**Sol.**

 d – d σ bonding

 p – d π bonding

 p – d π antibonding

 d – d σ antibonding

# PART C : MATHEMATICS

## SECTION – 1

### (Only One Option Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE option is correct.

**41.** The function y = f (x) is the solution of differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$ in (–1, 1) satisfying f (0) = 0. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

(A) $\frac{\pi}{3}-\frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ (C) $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

**Ans.** **[B]**

**Sol.** $\frac{dy}{dx}-\frac{x}{1-x^2}y=\frac{x^4+2x}{\sqrt{1-x^2}}$

$\text{I.F.} = e^{\int\frac{-x}{1-x^2}dx} = e^{\ln\sqrt{1-x^2}} = \sqrt{1-x^2}$

$y\sqrt{1-x^2}=\int(x^4+2x)dx=\frac{x^5}{5}+x^2+C$

$x = 0, y = 0$

$\therefore \quad C = 0$

$\frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}}$

$\therefore \quad \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}\frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}}dx = 0+2\int_0^{\frac{\sqrt{3}}{2}}\frac{x^2}{\sqrt{1-x^2}}dx$ ; put $x = \sin\theta$

$\therefore \quad I=\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ **Ans.**

**42.** The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\text{cosec}\,x)^{17}dx$ is equal to :

(A) $\int_0^{\log(1+\sqrt{2})}2(e^u+e^{-u})^{16}du$ (B) $\int_0^{\log(1+\sqrt{2})}(e^u+e^{-u})^{17}du$

(C) $\int_0^{\log(1+\sqrt{2})}(e^u-e^{-u})^{17}du$ (D) $\int_0^{\log(1+\sqrt{2})}2(e^u-e^{-u})^{16}du$

**Ans.** **[A]**

**Sol.** $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\,\text{cosec}\,x)^{17}\,dx$

Let cosec x + cot x = t

$\therefore$ cosec x – cot x $= \frac{1}{t}$

Hence 2 cosec x $= t + \frac{1}{t}$

Also (– cosec x cot x – $\text{cosec}^2$x) dx = dt

$\therefore$ – cosec x (cot x + cosec x)dx = dt

$\therefore$ $dx = \frac{-dt}{\text{cosec}x(\text{cot}x + \text{cosec}x)} = \frac{-2dt}{\left(t + \frac{1}{t}\right)(t)}$

Now $I = -2\int_{\sqrt{2}+1}^{1} \left(t + \frac{1}{t}\right)^{17} \frac{dt}{t\left(t + \frac{1}{t}\right)} = -2\int_{\sqrt{2}+1}^{1} \left(t + \frac{1}{t}\right)^{16} \frac{dt}{t}$

Put $t = e^u$

$\therefore$ $I = \int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16}\,du$

**43.** Coefficient of $x^{11}$ in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is

(A) 1051 (B) 1106 (C) 1113 (D) 1120

**Ans.** **[C]**

**Sol.** Coefficient of $x^{11}$ in $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$

$= (1 + {}^4C_1 x^2 + {}^4C_2 x^4 + {}^4C_3 x^6 + {}^4C_4 x^8)(1 + {}^7C_1 x^3 + {}^7C_2 x^6 + {}^7C_3 x^9)(1 + {}^{12}C_1 x^4 + {}^{12}C_2 x^8)$

Coefficient of $x^{11}$ is $1 \times {}^7C_1 \times {}^{12}C_2 + {}^4C_1 \times {}^7C_3 \times 1 + {}^4C_2 \times {}^7C_1 \times {}^{12}C_1 + {}^4C_4 \times {}^7C_1$

= 462 + 140 + 504 + 7 = 1113. **Ans.**

**44.** Let f : [0, 2] $\to$ R be a function which is continuous on [0, 2] and is differentiable on (0, 2) with f (0) = 1. Let $F(x) = \int_0^{x^2} f\left(\sqrt{t}\right)dt$ for $x \in [0, 2]$. If F'(x) = f ' (x) for all $x \in (0, 2)$, then F (2) equals

(A) $e^2 - 1$ (B) $e^4 - 1$ (C) e – 1 (D) $e^4$

**Ans.** **[B]**

**Sol.** F ' (x) = f (x) · 2x = f ' (x)

$\therefore$ $\frac{f'(x)}{f(x)} = 2x$

$\ln(f(x)) = x^2 + C$

f (0) = 1 $\Rightarrow$ C = 0

$f(x) = e^{x^2}$

$\therefore \quad F'(x) = 2xe^{x^2}$

Integrate $\Rightarrow \quad F(x) = e^{x^2}$

$F(0) = 0 \quad \Rightarrow \quad C = -1$

$F(x) = e^{x^2} - 1$

$F(2) = e^4 - 1$. **Ans.**

**45.** The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is :

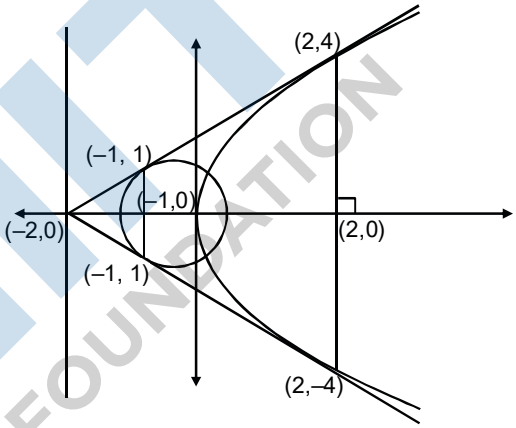
(A) 3 (B) 6 (C) 9 (D) 15

**Ans.** **[D]**

**Sol.** Tangent to $y^2 = 8x$ is $y = mx + \frac{2}{m}$ also touches $x^2 + y^2 = 2$

$\therefore \quad \left|\frac{\frac{2}{m}}{\sqrt{1+m^2}}\right| = \sqrt{2}$

$\therefore \quad m = \pm 1$




Tangent: $y = x + 2$ and $y = -x - 2$

$\therefore \quad$ point of intersection of tangents $(-2, 0) \equiv$ foot of directrix

Area $= 2\left(\frac{1}{2}(1+4)\times 3\right) = 15$. **Ans.**

**46.** For $x \in (0, \pi)$ the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

(A) infinitely many solutions (B) three solutions

(C) one solution (D) no solution

**Ans.** **[D]**

**Sol.** $\sin x + 2\sin 2x - \sin 3x = 3$

$\Rightarrow 2\sin 2x - 2\cos 2x \sin x = 3$

$\Rightarrow 2\cos 2x \sin x - 2\sin 2x + 1 + 1 + 1 = 0$

$\Rightarrow 2\cos 2x \sin x - 2\sin 2x + (\sin^2 x + \cos^2 x) + (\sin^2 2x + \cos^2 2x) + 1 = 0$

$\Rightarrow (1 - \sin 2x)^2 + (\sin x + \cos 2x)^2 + \cos^2 x = 0$, simultaneously all 3 are zero is not possible

$\therefore$ No solution

**47.** In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is

(A) $\frac{3y}{2x(x+c)}$ (B) $\frac{3y}{2c(x+c)}$ (C) $\frac{3y}{4x(x+c)}$ (D) $\frac{3y}{4c(x+c)}$

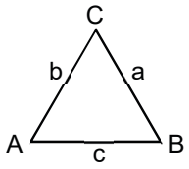
**Sol.** **[B]**

$a + b = x$

$ab = y$

$x^2 - c^2 = y$

$\frac{r}{R} = ?$




Now $\frac{r}{R} = \frac{\Delta}{s}\cdot\frac{4\Delta}{abc} = \frac{4\cancel{s}(s-a)(s-b)(s-c)}{\cancel{s}abc}$

$\frac{r}{R} = \frac{2(s-a)(s-b)\overbrace{(2s-2c)}^{a+b-c}}{yc} = \frac{2(s^2 - sx + y)(x-c)}{c \quad y}$

Now $x^2 - c^2 = y \Rightarrow (x-c)(x+c) = y$ .....(1)

$\therefore \quad \frac{r}{R} = \frac{2(s(s-x)+y)}{c}\frac{1}{x+c}$ using (1)

$= 2\left(\left(\frac{x+c}{2}\right)\left(\frac{c-x}{2}\right)+y\right)\frac{1}{c(x+c)} = 2\left(\frac{c^2-x^2}{4}+y\right)\frac{1}{c(x+c)}$

$= 2\left(y-\frac{y}{4}\right)\frac{1}{c(x+c)} = \frac{3y}{2c(x+c)}$ **Ans.**

**Aliter:** $a + b = x$

$ab = y$

$x^2 - c^2 = y$

$(a + b)^2 - c^2 = y \Rightarrow (a + b - c)(a + b + c) = ab$

$\Rightarrow \quad ab = 4s(s - c)$

$\Rightarrow \quad \frac{4\Delta}{\tan\frac{C}{2}} = ab \Rightarrow \tan\frac{C}{2} = \frac{c}{R}$

$\Rightarrow \quad \tan\frac{C}{2} = 2\sin C \Rightarrow \angle C = \frac{2\pi}{3}$

Now $\Delta = \frac{1}{2}ab\sin c \Rightarrow \Delta = \frac{\sqrt{3}ab}{4}$

Also $\Delta = \frac{abc}{4R}$

One equating both, we get $R = \frac{c}{\sqrt{3}}$

$r = \frac{\Delta}{s} \Rightarrow r = \frac{\sqrt{3}ab \cdot 2}{4(a+b+c)} \Rightarrow r = \frac{\sqrt{3}ab}{2(x+c)}$

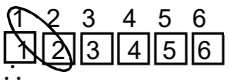
Now $\frac{r}{R} = \frac{3ab}{2c(x+c)} \Rightarrow \frac{r}{R} = \frac{3y}{2c(x+c)}$. **Ans.**

**48.** Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(A) 264 (B) 265 (C) 53 (D) 67

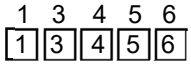
**Ans.** **[C]**

**Sol.**




$\therefore$ New structure

Card envelopes




Card envelopes

Case-I: 2 goes to 1

$\therefore$ number of way = D(4)

Case-II: 2 does not goes to 1,

$\therefore$ number of ways = D(5)

$\therefore$ answer = $D(4) + D(5) = 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)+5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$.

**49.** Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

**Ans.** **[A]**

**Sol.** possible cases

**CASE-I:** B G G B B

**CASE-II:** G G B B B

**CASE-III:** G B G B B

**CASE-IV:** B G B G B

**CASE-V:** G B B G B

Number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

$\frac{5 \times 3! \times 2!}{5!} = \frac{5 \times 6 \times 2}{120} = \frac{1}{2}$ **Ans.**

**50.** The quadratic equation p(x) = 0 with real coefficients has purely imaginary roots. Then the equation p(p(x)) = 0 has

(A) only purely imaginary roots (B) all real roots

(C) two real and two purely imaginary roots (D) neither real nor purely imaginary roots

**Ans.** **[D]**

**Sol.** Let $p(x) = Ax^2 + Bx + C$

If p(x) = 0 has purely imaginary roots $\Rightarrow$ sum of roots = 0

$\therefore$ B = 0

Also, D < 0

$\Rightarrow -4AC < 0 \Rightarrow AC > 0$

$p(x) = Ax^2 + C$

Now, $p(p(x)) = A\,(p(x))^2 + C = A\,(Ax^2 + C)^2 = A\,(A^2x^4 + 2ACx^2 + C^2) + C$

$\therefore\ p(p(x)) \equiv A^3x^4 + 2A^2Cx^2 + (AC^2 + C) = 0$

$$\Rightarrow\ x^2 = \frac{-2A^2C \pm \sqrt{4A^4C^2 - 4A^3(AC^2 + C)}}{2A^3}$$

$$x^2 = \left(\frac{-2A^2C \pm \sqrt{-4A^3C}}{2A^3}\right)$$

As, AC > 0

$\Rightarrow$ roots of p(p(x)) = 0 are neither real nor purely imaginary roots.

## SECTION - 2

### Comprehension Type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

### Paragraph For Questions 51 and 52

Let a, r, s, t be non zero real numbers. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point (2a, 0).

**51.** The value of r is

(A) $\frac{-1}{t}$ (B) $\frac{t^2 + 1}{t}$ (C) $\frac{1}{t}$ (D) $\frac{t^2 - 1}{t}$

**Ans.** **[D]**

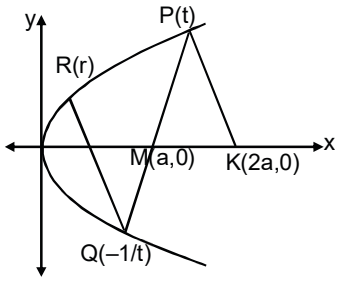
**52.** If st = 1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(A) $\frac{(t^2 + 1)^2}{2t^3}$ (B) $\frac{a(t^2 + 1)^2}{2t^3}$ (C) $\frac{a(t^2 + 1)^2}{t^3}$ (D) $\frac{a(t^2 + 2)^2}{t^3}$

**Ans.** **[B]**

**Sol.**

**(51)** **[D]**




$m_{QR} = m_{PK}$

$$\frac{2ar + \frac{2a}{t}}{ar^2 - \frac{a}{t^2}} = \frac{2at - 0}{at^2 - 2a}$$

$$\Rightarrow \quad t^2 - 2 = t\left(r - \frac{1}{t}\right);$$

$$\therefore \quad r = -\frac{t^2 - 1}{t} \quad \textbf{Ans.}$$

**(52)** **[B]**

Tangent at P is

$$yt = x + at^2 \qquad .....(1)$$

Normal at S $(as^2, 2as)$ is $y + sx + 2as + as^3$

Put $s = \frac{1}{t} \quad \Rightarrow \quad y + \frac{x}{t} = \frac{2a}{t} + \frac{a}{t^3}$ .....(2)

from (1) and (2), eliminate x, we get

$$y = \frac{a(t^2 + 1)^2}{2t^3}. \quad \textbf{Ans.}$$

**Paragraph For Questions 53 and 54**

Given that for each $a \in (0,1)$, $\lim\limits_{h \to 0^+} \int\limits_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$ exists. Let this limit be g(a). In addition, it is given that the function g(a) is differentiable on (0,1).

**53.** The value of $g\left(\frac{1}{2}\right)$ is

(A*) $\pi$ (B) $2\pi$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

**54.** The value of $g'\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{2}$ (B) $\pi$ (C) $\frac{-\pi}{2}$ (D*) 0

**Sol.** (i) $g(a) = \int\limits_0^1 t^{-a}(1-t)^{a-1}\,dt$ .....(1)

Using King $g(a) = \int\limits_0^1 (1-t)^{-a}(t)^{a-1}\,dt$ .....(2)

Now g (a) = g (1 – a)

Differentiating with respect to a

g ' (a) = – g ' (1 – a)

Put a = $\frac{1}{2}$ ; $\therefore$ g ' $\left(\frac{1}{2}\right)$ = 0 **Ans.**

**Aliter:** g (a) = $\int_0^1 t^{-a}(1-t)^{a-1}\,dt$

differentiate under the sign of integral w.r.t. a keeping t constant

g ' (a) = $\int_0^1 t^{-a}(1-t)^{a-1}\ln(1-t) - (1-t)^{a-1}\ln t)dt$

Put a = $\frac{1}{2}$

I = g' $\left(\frac{1}{2}\right)$ = $\int_0^1 \frac{\ln(1-t) - \ln t}{\sqrt{t-t^2}}dt$

Using King I = – I

$\therefore$ I = 0 **Ans.**

**Paragraph For Questions 55 and 56**

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let $x_i$ be the number on the card drawn from the $i^{th}$ box, i = 1, 2, 3.

**55.** The probability that $x_1 + x_2 + x_3$ is odd, is

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

**Ans.** **[B]**

**56.** The probability that $x_1$, $x_2$, $x_3$ are in an arithmetic progression, is

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

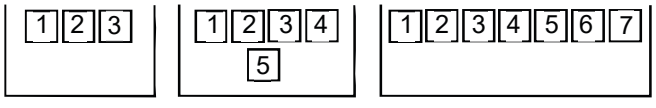
**Ans.** **[C]**

**Sol.**

**(55)**

$x_1 + x_2 + x_3$

| | | |
|---|---|---|
| I | 0 + 0 + 0 = 0 | |
| | 0 0 e = e | |
| II | 0 e e = 0 | |
| | e e e = e | |



I[st]

$x_1$ 2 possibilities

$x_2$ 3 possibilities

$x_3$ 4 possibilities

Total = $2 \cdot 3 \cdot 4 = 24$

II[st]

0 e e = $2 \cdot 2 \cdot 3 = 12$

e 0 e = $1 \cdot 3 \cdot 3 = 9$

e e 0 = $1 \cdot 2 \cdot 4 = 8$

Total = 29

Total = 29 + 24 = 53

$\frac{53}{3 \cdot 5 \cdot 7} = \frac{53}{105}$ **Ans.**

**(56)** [C]

$x_1, x_2, x_3$

$2x_2 = x_1 + x_3$

2 = 2 × 1 = 1 + 1

4 = 2 × 2 = 1 + 3 / 2 + 2 / 3 + 1

6 = 2 × 3 = 1 + 5 / 2 + 4 / 3 + 3

8 = 2 × 4 = 1 + 7 / 2 + 6 / 3 + 5

10 = 2 × 5 = 3 + 7

Total cases = 11

probability $= \frac{11}{105}$.

## SECTION-3

### Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D), out of which one is correct.

**57.** Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; k = 1, 2, ……..,9.

| | **List-I** | **List-II** | |
|---|---|---|---|
| P. | For each $z_k$ there exists a $z_j$ such that $z_k \cdot z_j = 1$ | 1. | True |
| Q. | There exists a $k \in \{1, 2, ……., 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers | 2. | False |

R. $\frac{|1-z_1||1-z_2|.....|1-z_9|}{10}$ equals 3. 1

S. $1-\sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals 4. 2

**Codes:**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 1 | 2 | 4 | 3 |
| (B) | 2 | 1 | 3 | 4 |
| (C) | 1 | 2 | 3 | 4 |
| (D) | 2 | 1 | 4 | 3 |

**Sol.** [C]

$z^{10} = 1$ where $z \neq 1$

P. True

S. $1 - \sum_{k=1}^{9}\cos\left(\frac{2\pi k}{10}\right) = 1 - (-1) = 2.$

**58.**

| | **List-I** | **List-II** |
|---|---|---|
| P. | The number of polynomials f(x) with non-negative integer coefficients of degree $\leq 2$, satisfying f(0) = 0 and $\int_0^1 f(x)dx = 1$, is | 1. 8 |
| | The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is | 2. 2 |
| R. | $\int_{-2}^{2}\frac{3x^2}{(1+e^x)}dx$ equals | 3. 4 |
| S. | $\frac{\left(\int_{\frac{-1}{2}}^{\frac{1}{2}}\cos 2x \log\left(\frac{1+x}{1-x}\right)dx\right)}{\left(\int_{0}^{\frac{1}{2}}\cos 2x \log\left(\frac{1+x}{1-x}\right)dx\right)}$ equals | 4. 0 |

**Codes:**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 3 | 2 | 4 | 1 |
| (B) | 2 | 3 | 4 | 1 |
| (C) | 3 | 2 | 1 | 4 |
| (D*) | 2 | 3 | 1 | 4 |

**Sol.** P. $f(x) = ax^2 + bx$

Now $\int_0^1 f(x)dx = 1 \Rightarrow 2a + 3b = b$

$\therefore$ $a = 0 \Rightarrow b = 2$

or $b = 0 \Rightarrow a = 3$

So, $f(x) = 2x$ ; $f(x) = 3x^2$

$$R.I = \int_{-2}^{2} \frac{3x^2}{(1+e^x)}dx = \int_0^2 \left(\frac{3x^2}{1+e^x} + \frac{3x^2}{1+e^{-x}}\right)dx = \int_0^2 3x^2dx = 8$$

**59.**

| | List-I | List-II |
|---|---|---|
| P. | Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals | 1. 1 |
| Q. | Let $A_1, A_2, \ldots\ldots, A_n$ ($n > 2$) be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a_k}$ be the position vector of the point $A_k$, $k = 1, 2, \ldots\ldots, n$. If $\left\lvert\sum_{k=1}^{n-1}(\vec{a_k} \times \vec{a_{k+1}})\right\rvert = \left\lvert\sum_{k=1}^{n-1}(\vec{a_k} \cdot \vec{a_{k+1}})\right\rvert$, then the minimum value of n is | 2. 2 |
| R. | If the normal from the point P (h, 1) on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is | 3. 8 |
| S. | Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is | 4. 9 |

**Codes:**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 4 | 3 | 2 | 1 |
| (B) | 2 | 4 | 3 | 1 |
| (C) | 4 | 3 | 1 | 2 |
| (D) | 2 | 4 | 1 | 3 |

**Sol.** **[A]**

**P.** $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\} = 9$

**R.** $h = 2$

**60.** Let $f_1 : R \to R$, $f_2 : [0, \infty) \to R$, $f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x|, & \text{if } x < 0 \\ e^x, & \text{if } x \geq 0 \end{cases}$$

$$f_2(x) = x^2,$$

$$f_3(x) = \begin{cases} \sin x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

and

$$f_4(x) = \begin{cases} f_2(f_1(x)), & \text{if } x < 0 \\ f_2(f_1(x)) - 1, & \text{if } x \geq 0 \end{cases}$$

| | **List-I** | | **List-II** |
|---|---|---|---|
| P. | $f_4$ is | 1. | onto but not one-one |
| | $f_3$ is | 2. | neither continuous nor one-one |
| R. | $f_2 o f_1$ is | 3. | differentiable but not one-one |
| S. | $f_2$ is | 4. | continuous and one-one |

**Codes:**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 3 | 1 | 4 | 2 |
| (B) | 1 | 3 | 4 | 2 |
| (C) | 3 | 1 | 2 | 4 |
| (D) | 1 | 3 | 2 | 4 |

**Ans.** **[D]**

**Sol.** $$f_1(x) = \begin{cases} -x, & \text{if } x < 0 \\ e^x, & \text{if } x \geq 0 \end{cases}$$

$$f_2(x) = x^2,$$

$$f_3(x) = \begin{cases} \sin x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

and

$$f_4(x) = \begin{cases} f_2(f_1(x)), & \text{if } x < 0 \\ f_2(f_1(x)) - 1, & \text{if } x \geq 0 \end{cases}$$

Now verify.